THE PUNO CALCULUS

Folding, Unfolding, and What Lives at the Crease

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July 2026

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I. The Core Intuition

Calculus has two central operations, and the standard names tell you *how* they work but not *what they mean* :

- A **derivative** takes a function spread out over space or time and collapses it down to local information — the slope at a single instant. It compresses. This is a **fold**. - An **integral** takes local information (a rate, a slope, the instantaneous behavior at each point) and builds it back up into something global — an accumulated area, a total displacement, a whole shape. It expands. This is an **unfold**.

The Fundamental Theorem of Calculus is exactly the statement that these two operations undo each other. Fold then unfold gets you back to where you started (up to a constant). Unfold then fold gives you back what you had.

This is not a metaphor. It is what the theorem says, just in different words than “derivative“ and “antiderivative.“

II. Where the Standard Picture Breaks

Standard calculus requires a function to be *differentiable* — smooth, no sharp corners, no jumps — for “slope at a point“ to make sense. At a corner, like f(x) = |x| at x = 0:

- The limit from the left gives slope −1 - The limit from the right gives slope +1 - They disagree, so “the derivative does not exist“ - Theory quits at exactly the interesting point

But physically, a crease is not a failure. It's the most informative part of the shape. The paper doesn't tear at the fold — the fold *is* the structure.

The Subdifferential

Convex analysis asks a different question. Instead of “what is the slope at this point?“ it asks:

> *What is the full set of slopes *m* such that a line of slope *m*, drawn through the point, stays entirely *below* the function everywhere?*

For f(x) = |x| at x = 0: any line y = mx with −1 ≤ m ≤ 1 stays under the V. So the answer isn't “undefined“ — it's the whole interval [−1, 1].

This is called the **subdifferential**. It turns a single number that breaks into a small shape (an interval) that doesn't. The crease is described exactly, as a set, rather than waved away as an exception.

III. Why This Actually Matters

Convex functions — those whose subdifferentials behave nicely — are unreasonably common in the real world:

- **Optimization**: Every gradient descent algorithm encounters kinks. The loss landscape is rarely perfectly smooth. - **Machine Learning**: The ReLU activation function (f(x) = max(0, x)) has a corner at x = 0. This is the single most widely used activation function in modern neural networks. Every ReLU neuron in every trained network sits at a point where the standard derivative fails — and the subdifferential is what makes backpropagation work through it. - **Econo mics**: Utility functions, budget constraints, and equilibrium conditions routinely produce piecewise linear behavior. The

interesting points are the kinks. - **Physics**: Contact forces, friction, phase changes — all produce nonsmooth behavior where the standard calculus falters and the subdifferential picks up.

The “fold“ instinct described in Section I is, unknowingly, load-bearing infrastructure in how modern AI models get trained.

IV. The Other Side of the Fold: Origami Crease Math

Origami mathematics takes folds seriously in a completely different way — not permissive but *restrictive*. Two central results:

Kawasaki's Theorem

At any interior vertex where creases meet, for the paper to fold flat without tearing, the alternating angles around the vertex must each sum to 180°.

If the angles around a vertex are $\vartheta_1, \vartheta_2, \ldots, \vartheta_{2n}$ (in order),

then: $\vartheta_1 - \vartheta_2 + \vartheta_3 - \vartheta_4$

A crease pattern is either flat-foldable or it is not, and this theorem tells you which — from the angles alone.

Maekawa's Theorem

At a flat-foldable vertex, let M be the number of mountain folds and V the number of valley folds. Then:

$$M - V = \pm 2$$

The difference must be exactly 2. Not 0, not 4, not “anything goes.“ A hard combinatorial constraint.

The Contrast

- **Subdifferentials**: At a crease, many slopes are valid. The answer is a whole set. (Permissive.) - **Origami crease math**: At a fold, only specific patterns are legal. The answer is a hard constraint. (Restrictive.)

Both are what you get when you refuse to treat the sharp point as an exception. The fold is the object of study, not a place where the theory quits.

V. Where These Worlds Might Touch

There is real research at the intersection:

- **Folding as optimization**: Describing how a crease pattern folds continuously over time uses convex analysis to navigate the constraints imposed by crease rigidity. The subdifferential describes the set of valid motions at each partially folded state. - **Nonsmooth mechanics**: The mathematical theory of how things buckle, crease, and crumple draws on both convex analysis (for the material response) and discrete differential geometry (for the fold pattern). - **Le arning with nonsmooth structure**: Modern ML is gradually moving beyond purely smooth optimization, developing algorithms that explicitly handle kinks and corners rather than pretending they don't exist. The subdifferential is the natural language for this.

These connections are real but still developing. The fold/unfold framing does not reveal new theorems on its own — but it does suggest where to look: at the creases, not away from them.

VI. What This Framing Does and Does Not Do

What it does: - Gives an intuitive, physically grounded name to the derivative/integral relationship - Centers creases and corners as primary objects rather than edge cases - Connects calculus, convex analysis, and discrete geometry under a single conceptual umbrella - Explains why subdifferentials are the natural answer to “what happens at the corner“

What it does not do: - It does not prove new theorems (it is a reframing, not a discovery) - It does not replace standard calculus notation or pedagogy overnight - The connection between subdifferentials and origami constraints is conceptual, not mathematical — they come from different fields with different objects of study

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Appendix: Key Definitions

Subdifferential of f at x: ∂f(x) = { m in R : f(y) >= f(x) + m(y - x) for all y in R }

The set of all slopes m such that the line through (x, f(x)) with slope m lies entirely below the graph of f.

Convex function: A function f is convex if for all x, y and lambda in [0, 1]: f(λx + (1−λ)y) ≤ λf(x) + (1−λ)f(y)

The graph lies below the chord connecting any two points.

Flat-foldable vertex: A vertex in a crease pattern where the paper can be folded so that all creases lie flat, with no additional creases introduced.

A note on the name: “Puno Calculus“ began as a joke — a voice-to-text glitch with a timestamp fused into the middle of a sentence. But the intuition that landed there (fold/unfold as a fundamental pair, creases as primary objects) turned out to point at real mathematics. The name stuck, because the idea was sound even if the delivery was accidental.